© *Proc. Indian natn. Sci. Acad.*, **49**, A, No. 2, 1983, pp. 325–332.
Printed in India.

Semiconductors

THE EFFECT OF MAGNETIC FIELD ON THE ABSORPTION COEFFICIENT OF HOT ELECTRONS IN SEMICONDUCTORS

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(*Received 12 May 1982*)

Expressions are derived for the absorption coefficient of light by electrons in non-degenerate Semiconductors at low temperatures ($T << W_0$, W_0 is the energy of optical phonon) in the presence of a strong d.c. electric field and a weak magnetic field using the distribution function of the carriers. The inelastic scattering of electrons by optical phonons has been considered to be the dominant scattering mechanism. The results have been compared with earlier theoretical results in the last section.

Keywords : Magnetic Field; Absorption Coefficient; Hot Electrons; Semiconductors

INTRODUCTION

THE study of hot-electron transport and optical phenomena in semiconductors are of great interest because they form an important source of information concerning the process of scattering energy and momentum of charge carriers. The reason is, first of all, that the form of the nonequilibrium stationary distribution function depends considerably on the scattering mechanisms. In the previous papers (Bonch-Bruivich and El Sharnouby (1972), El Sharnouby (1972, 1982), an analytical expression for the absorption coefficient of hot electrons has been obtained without applying a magnetic field. The elastic scattering of electrons by acoustic phonons was considered (El Sharnouby, 1972). The elastic scattering of electrons by impurities and optical phonons was taken into account (El Sharnouby, 1982). Since, at high electric field the scattering of electrons are considered to be elastic and isotropic (the drift velocity was considered to be small compared to the r.m.s. velocity of the chaotic motion), the distribution function f (p) is nearly isotropic and can be put as a sum of a symmetric (fs) and antisymmetric (fa) parts such that fa $\ll$ fs.

For semiconductors of strong interaction the carriers with optical phonons of energy (w_0) such as in *p-Ge*, there exist, at low temperatures ($T \ll w_0$), an interval of electric field strengths are restricted by the condition $\bar{E}_0 \ll E_0 \ll E_0^+$ in which the scattering with optical phonons will be inelastic an anisotropic. The distribution function f (**p**), for this interval of field becomes anisotropic, and can be expressed in terms of Dirac δ-function (Baroff, 1964)

$$f(\mathbf{p}) = 2\psi(\epsilon)^{s} (\cos\theta - 1) \qquad \ldots(1)$$

where ϵ is the energy of the electron of momentum $\mathbf{p}$, θ is the angle between the momentum and field. In absence of magnetic field H, the dependence of absorption coefficient on d.c. electric field E_0 and on frequency of light W, in case of inelastic scattering with optical phonons (anisotropic distribution function) was studied (Bonch-Bruivich & El Sharnouby, 1972). The purpose of this paper is to study the dependence of the absorption coefficient of light on E_0 and W in presence of magnetic field H. In the next section, the Boltzmann transport equation has been discussed and solved to obtain the distribution function of hot carriers.

The Transport Equation

Consider a semiconductor sample of one type of carriers at low temperatures ($T \ll w_0$), subjected to a high d.c. electric field $\mathbf{E}_0$, small parallel a.c. electric field $\mathbf{E}_1 e^{-iwt}$ and a perpendicular magnetic field $\mathbf{H}$.

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 e^{-iwt}, \; E_1 \ll E_0 \qquad \ldots(2)$$

let

$$f(\mathbf{p}, t) = f_0(\mathbf{p}) + f_1(\mathbf{p}) e^{-iwt}, \; f_1(\mathbf{p}) \ll f_0(\mathbf{p}) \qquad \ldots(3)$$

Suppose $H = 0$ and the electric field strengths are restricted by the condition $E_0^- \ll E_0 \ll E_0^+$, where the characteristic field strengths E_0^+ and E_0^- are determined from the relation

$$eE_0^{\pm} \tau^{\pm} = P_0 \qquad \ldots(4)$$

Here e — charge of the electron, P_0 — momentum of the electron with energy $\epsilon = W_0$, τ^+ and τ^- — relaxation times corresponding to the emission and absorption optical phonon (Conwell, 1967). Since $\tau^- \sim e^{W_0/T} \tau^+ \gg \tau^+$, then $E^- \ll E_0^+$. For simplicity we consider that at $E_0 = 0$, the electrons exist in a sphere of radius $= P_T$ (P_T — thermal momentum; $P_T \ll P_0$). In case of $E_0 \ll E_0^-$, most of the electrons absorb optical phonons, do not reach the boundary of "active" region ($\epsilon(p) > W_0$), this absorption leads to spontaneous emission of optical phonons. The scattering in this case is nearly elastic and the distribution function is isotropic. For $E_0 \gg E_0^-$, acceleration of an electron from $\epsilon = 0$ to $\epsilon = w_0$ is not interrupted by elastic scattering with phonon, since it reaches the boundary of "active" region in time $\tau_E = P_0/eE_0 \ll \tau^-$ and deepen into this region with quantity $\Delta P \sim eE_0 \tau^+$, then the electron emits an optical phonon and stops moving, returning to a sphere of radius ΔP ($\Delta P < P_0$) in the "passive" ($\epsilon(\mathbf{p}) < w_0$) region and the cycle is repeated (Fig. 1). In such fields all electrons possess velocities directed along the field and the distribution has the form (I). For $E_0 \gg E_0^+$, the scattering of electrons with optical phonons is elastic and the distribution is isotropic. The condition $T \ll W_0$ means that most of the electrons are in the "passive" region.

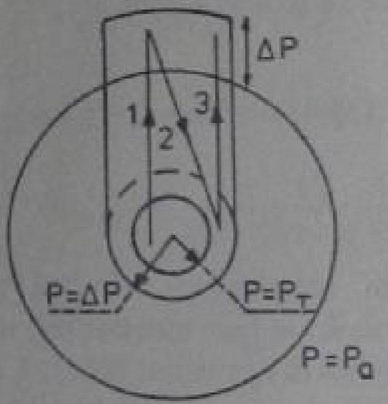

FIG. 1. Motion of electron in momentum space for $E_0^- << E_0 << E_0^+$ 1-Trajectory of initial acceleration; 2-After emission optical phonon; 3-Trajectory of 2nd cycle.

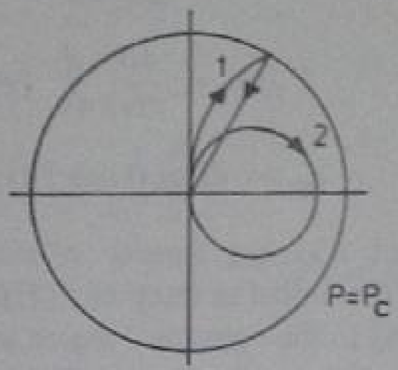

FIG. 2. Motion of electron in presence of magnetic field (for simplicity assuming $\Delta P = 0$; $T = 0$) : 1-Trajectory for $H < H_C$; 2-$H > H_C$.

When a perpendicular magnetic field **H** is switched on, the trajectory of the electron is bended (for parabalic band, the trajectory is a circular arc of redius mCE_0/H). In case of $H < H_c$, ($H_c = 2c/V_0E_0$), trajectory reaches the boundry of "active" region and for $H > H_c$, the trajectory is closed in "passive" region (Fig. 2).

If the equation of the trajectory in polar coordinates ϵ, θ, ϕ with polar axis 11^3E_0 is $\phi = 0$; $\theta = \theta(\epsilon)$, then for $H < H_c$, the distribution function is different from zero along the trajectory and can be written in the form (Vosylius & Levinson, 1966, 1967)

$$f(\mathbf{p}) = 4\,\Pi\,\delta(\phi)\,\psi(\epsilon)\,\delta\,(\cos\theta - \cos\theta\,(\epsilon)) \qquad \ldots(5)$$

From eqns. (2), (3) and (5) we have

$$f(\mathbf{p}) = 4\,\Pi\,\delta(\phi)\,\psi_0(\epsilon)\,\delta\,(\cos\theta\cos\theta)\,(\epsilon)) \qquad \ldots(6)$$

$$f_1(\mathbf{p}) = 4\,\Pi\,\delta(\phi)\,\psi_1(\epsilon)\,\delta\,(\cos\theta - \cos\theta\,(\epsilon))$$

Since $f_1(\mathrm{p}) \ll f_0\,(\mathrm{p})$,

The Boltzmann's equation for $E_0^- \ll E_0 \ll E_0^+$ and $H < H_c$ has the form

$$F_{00}(\mathbf{p}) = J\,[f_0\,(p)] \qquad \ldots(7)$$

$$iwf_1(\mathbf{p}) + F_{10}(\mathbf{p}) + F_{10}(\mathbf{p}) = J[f_1(\mathbf{p})] \qquad \ldots(8)$$

Where
$$F_{00}(\mathbf{p}) = \frac{4\Pi\, d}{g(\epsilon) d} \cdot [g(\epsilon)\psi_0(\epsilon)\ \epsilon'_{F_0}(\epsilon)]\ \delta(\phi)\ \delta(\cos\theta - \cos\theta(\epsilon))$$

$$F_{10}(\mathbf{p}) = \frac{4\Pi}{g(\epsilon)}\frac{d}{d\epsilon}\ [g(\epsilon)\ \psi_0(\epsilon)\ \epsilon'_{F_1}(\epsilon)]\ \delta(\phi)\ \delta(\cos\theta - \cos\theta(\epsilon))$$

$$F_{01}(\mathbf{p}) = \frac{4\Pi}{g(\epsilon)}\frac{d}{d\epsilon}\ [g(\epsilon)\psi_1(\epsilon)\ \epsilon'_{F_0}(\epsilon)]\ \delta(\phi)\ \delta(\cos\theta - \cos\theta(\epsilon)) \quad ...(9)$$

$$\epsilon'_{Fi} = eE_i v(\epsilon)\cos\theta(\epsilon),\ (i = 0,1) \quad ...(10)$$

$g(\epsilon)$ and $v(\epsilon)$ are density of states and velocity of the electron. In the "passive" region, the collision integral $J[f(\mathbf{p})] = I_0\,\delta(\mathbf{p})$, where I_0 is the number of electrons returning to the "passive" region in a unit time (Vosylius & Levinson, 1966). Considering $\mathbf{P} \neq 0$, we can take $I_0\,\delta(\mathbf{P}) = 0$. In the "active" region, the collision integral $J[f(P)] = \frac{-f(\mathbf{p})}{\tau(\epsilon)}$, where $\tau^{-1}(\epsilon)$ is the probability of emission optical phonon in unit time by an electron of energy $\epsilon \geqslant W_0$ (Conwell, 1967)

$$\tau(\epsilon) = \tau_0(y-1)^{-1/2},\ y = \frac{\epsilon}{W_0},\ \tau_0 = \frac{\Pi\sqrt{2}\,\hbar^2\rho\sqrt{W_0}}{D^2 m^{3/2}},\ \epsilon = \frac{P^2}{2m} \quad ...(11)$$

According to the above consideration, the Boltzmann's equation has the form :

In "passive" region ($\epsilon < W_0$)

$$F_{00}(\mathbf{p}) = 0 \quad ...(12)$$

$$-iwf_1(\mathbf{p}) + F_{10}(\mathbf{p}) + F_{01}(\mathbf{p}) = 0 \quad ...(13)$$

In "active" region ($\epsilon \geqslant W_0$)

$$\frac{1}{g(\epsilon)}\frac{d}{d\epsilon}(g(\epsilon)\,\psi_0(\epsilon)\epsilon'_{F_0}(\epsilon)) = -\frac{\psi_0(\epsilon)}{\tau(\epsilon)} \quad ...(14)$$

$$-iw\,\psi_1(\epsilon) + \frac{1}{g(\epsilon)}\frac{d}{d\epsilon}\ (g(\epsilon)\ \psi_0(\epsilon)\ \epsilon'_{F_1}(\epsilon))$$
$$+\frac{1}{g(\epsilon)}\frac{d}{d\epsilon}\ (g(\epsilon)\ \psi_1(\epsilon)\ \epsilon'_{F_0}(\epsilon)) = -\frac{\psi_1(\epsilon)}{\tau(\epsilon)} \quad ...(15)$$

To find $\psi_0(\epsilon)$ and $\psi_1(\epsilon)$, it is necessary to solve the system of eqns. (12)–(15).
From eqns. (9) and (12), we have

$$\psi_0(\epsilon) = \frac{C_0}{g(\epsilon)\ v(\epsilon)\cos\theta(\epsilon)},\ (\epsilon < W_0) \quad ...(16)$$

From eqn. (14),

$$\psi_0(\epsilon) = \frac{C_1}{g(\epsilon)\ v(\epsilon)\cos\theta(\epsilon)}\ \exp\left[-\int_{W_0}^{\epsilon}\frac{dW}{\tau(W)\ \epsilon'_{F_0}(W)}\right]$$

Since ψ_0 is continuous at $\epsilon = W_0$, we have $C_1 = C_0$

C_0 is determined from the normalization condition

$$\int_0^\infty g(\epsilon)\, \psi_0(\epsilon)\, d\epsilon = n \qquad ...(17)$$

Using eqns. (10) and (11), we have

$$\psi_0(y) = \frac{C_0}{g(y)\, v(y) \cos\theta\,(y)} \begin{cases} 1 & , \quad y < 1 \\ \exp[-\lambda(y-1)^{3/2}], & y \geqslant 1 \end{cases} \qquad ...(18)$$

where $v(y) = \sqrt{\frac{2W_0\, y}{m}}$, $g(y) = \frac{m\sqrt{2mwoy}}{2\Pi^2\, \hbar^3}$, $\cos\theta(y) = \sqrt{1 - h^2\, y}$, $\hbar$-Planck constant, $h = H/H_c < 1$ and $y = \epsilon/W_0$.

λ is a large parameter given by $\lambda = \frac{P_0}{3e\, E_0\tau_0\, \sqrt{1-h^2}} = \frac{\tau_{E0}}{3\tau_0\sqrt{1-h^2}} \gg 1$

$$...(19)$$

$$C_0 = \frac{n}{\left[P_0 \frac{\text{arc sin } h}{h} + \frac{P_0\, \lambda^{-2/3}\, \Gamma(5/3)}{2\sqrt{1-h^2}}\right]} \qquad ...(20)$$

If we neglect the second term in the denominator of C_0 in (20), the distribution function ψ_0 (ϵ) becomes that obtained (Vosylius & Levinson, 1966) using eqns. (13)–(16), and applying the condition of continuity of $\psi_1(\epsilon)$ at $\epsilon = W_0$, we have

$$\psi_1(y) = \frac{K}{g(y)\, v(y) \cos\theta(y)} \exp\,[iwt_h(y)],\ (y < 1) \qquad ...(21)$$

For $y = 1$

$$\psi_1(y) = \left[\frac{K + \lambda C_0 E_1/E_0(y-1)^{3/2} \exp\,(-it_h)}{g(y)\, v(y) \cos\theta(y)}\right] \exp\,[it_h(y) - \lambda(y-1)^{3/2}]$$

$$...(22)$$

where $t_h(y) = w\tau_{E_0} \frac{\text{arc sin } h\sqrt{y}}{h}$, $t_h = t_h(y = 1)$

and K is a complex constant which is determined, from the condition.

$$\int_0^\infty g(\epsilon)\, \psi_1(\epsilon)\, d\,\epsilon = 0$$

as $$K = \frac{-2\, E_1 C_0}{3\, E_0}\, [(\cos t_h + \beta t_h^{-1} \sin t_h) +$$

$$i\{\sin t_h + \beta\, t_h^{-1}\, (1 - \cos t_h)\}\,]^{-1} \qquad ...(23)$$

where $$\beta = \frac{t_0 \lambda^{-2/3} \Gamma(5/3)}{2t_h \sqrt{1-h^2}}, \; t_0 = \omega \tau_{E0}$$

and Γ is a Gamma function

The Current Density and the Absorption Coefficient

The longitudinal (dissipative) component of the current density in a semicondnctor is given by

$$j_1 = e\sqrt{\frac{2}{m}} \int_0^\infty \sqrt{\epsilon}\, g(\epsilon)\, [\psi_0(\epsilon) + \psi_1(\epsilon)\, e^{-i\omega t}]\, d\epsilon \qquad \ldots(24)$$

From eqn. (18), we have

$$j_{10} = e\sqrt{\frac{2}{m}} \int_0^\infty \sqrt{\epsilon}\, g(\epsilon)\, \psi_0(\epsilon)\, d\epsilon$$

$$= eC_0 W_0 \left[-\frac{2}{h^2} (\sqrt{1-h^2} - 1) + \frac{\lambda^{-2/3} \Gamma(5/3)}{\sqrt{1-h^2}} \right]$$

Since $h < 1$, using eqn. (20), we obtain the following expression for j_{10} :

$$j_{10} = \frac{new_0 h}{P_0 \arcsin h} [1 + 1/4h^2 + \left(1 - \frac{h}{2 \arcsin h}\right) \lambda_0^{-2/3} \Gamma(5/3)] \qquad \ldots(25)$$

where $\lambda_0 = \frac{\tau_{E0}}{\tau_0} \gg 1.$

Neglecting the 2*nd* and 3*rd* terms in the square bracket of eqn. (25), we have the following result as obtained (Vosylius & Levinson, 1966)

$$J_{1_0} \simeq \frac{new_0 h}{P_0 \arcsin h} = J_0 \frac{h}{\arcsin h} \qquad \ldots(26)$$

where $J_0 = 1/2\, nev_0$ is the saturation current following at $H = 0$. The a.c longitudinal component of the current density has the form

$$J_{11} = e\sqrt{\frac{2}{m}}\, e^{-i\omega t} \int_0^\infty \sqrt{\epsilon}\, g(\epsilon)\, \psi_1(\epsilon)\, d\epsilon \qquad \ldots(27)$$

Using eqns. (21) and (22), we obtain

$$J_{11} = ew_0\, e^{-i\omega t} \left[\frac{2k}{t_0^2 - h^2} \sqrt{1-h^2}\, e^{it} h - 1 - it_0 e^{it} h) \right.$$
$$\left. + \frac{K \lambda^{-2/3} \Gamma(5/3)\, e^{it} h}{\sqrt{1-h^2}} + \frac{2C_0 F_1}{3F_0} \lambda^{-2/3} \Gamma(5/3) \right] \qquad \ldots(28)$$

Since the high frequency conductivity $\delta_1(w, E_0, H)$ is expressed in terms of J_{11} by the form

$$J_{11} = \delta_1(w, E_0, H)\, E_1\, e^{-iwt}. \qquad ...(29)$$

We get, taking into account of eqns. (23), (28) and (29), an expression for the real part of $\delta_1(w, E_0, H)$ as

$$\text{Re}\ \delta_1(W, E_0, H) = \frac{4eC_0W_0}{3E_0} I\,[I_1(1 - \cos t_h) + I_2 \sin t_h + I_3(1 - \sqrt{1 - h^2})\,] \qquad ...(30)$$

where

$$I = [(t_0^2 - h^2)\,\{\,1 + 2\,\beta^2 t_h^{-2}\,(1 - \cos t_h) + 2\beta t_h^{-1}\ \sin t_h\}]^{-1} \qquad ...(31)$$

$$I_1 = \frac{2\beta\sqrt{1 - h^2}\,(t_0^2 - h^2)}{t_0 t_h} - \frac{\beta t_0}{t_h} - 1$$

$$I_2 = \frac{t_0^2 - h^2}{t_0}(2\sqrt{1 - h^2} - 1) \sin t_h$$

$$I_3 = I + \frac{\beta \sin t_h}{t_h} - \frac{t_0^2 - h^2}{t_0 \beta} \qquad ...(32)$$

In case $h = 0$, taking into account of eqns. (20), we have

$$\text{Re}\ \delta_1(w, E_0, 0) = \tfrac{2}{3} \frac{neP_0}{mE_0} \left[\frac{(\beta - 1)(1 - \cos t_0) + t_0 \sin t_0}{t_0^2 + 2\,\beta^2(1 - \cos t_0) + 2\beta^t{}_0 \sin t_0} \right] \qquad ...(33)$$

which is the same as was obtained (Bonch-Bruivich & El Sharnouby, 1972). The absorption coefficient is expressed in terms of $\text{Re}\ \delta(w, E_0, H)$ as

$$\alpha = \frac{4\Pi}{c\sqrt{\epsilon_0}} \text{Re}\ \delta_1(w, E_0, H) \qquad ...(34)$$

where c is the velocity of light in vacuum, ϵ_0 is the dielectric constant.

Comparison with Existing Theoretical Results and Discussion

Comparing eqns. (25) and (26), the value of J_{10} obtained in this paper is a better approximation than that obtained (Vosylius & Levinson, 1966). From eqns. (30), (31), (32) and (34), it is obvious that in the presence of magnetic field, we have

$$\alpha(W, E_0, H)_{t_0 \to h} \to \infty$$

From eqns. (31), for $t_0 \gg h$, we have

$$I = \left[(t_0^2 - h^2) \left\{ \left(\cos t_h + \frac{\beta \sin t_h)^2}{t_h} \right) + \left(\sin t_h + \frac{\beta(1 - \cos t_h)^2}{t_h} \right) \right\} \right]^{-1} > 0$$

According to eqns. (30) and (32), we obtain for $h < 1$,

$$\begin{aligned} L &= I_1(1 - \cos t_h) + I_2 \sin t_h + I_3(1 - \sqrt{1 - h^2}) \\ &= \beta_1 (1 - \cos t_h) + \beta_2 \sin t_h + \beta_3 \end{aligned} \quad \ldots(35)$$

Where $\beta_1 = \dfrac{2\lambda^{2/3}}{\Gamma(5/3)} - \dfrac{3h^2\lambda^{2/3}}{\Gamma(5/3)} - 1$

$$\beta_2 = t_0 + \frac{\lambda^{2/3} h^2}{t_0 \Gamma(5/3)}$$

and $\beta_3 = \frac{1}{2} h^2 (1 - 1/2\, \lambda^{2/3} \Gamma(5/3)\, t_0^2)$

It is obvious that L, and hence the corresponding $\alpha(W, E_0, H)$, changes its sign for the value of t_h restricted by the condition

$$2\Pi(21 + 1)\,[1 - \gamma] < t_h < 2\Pi(21 + 1) \quad \ldots(36)$$

where

$$\gamma = \frac{h\Gamma(5/3)}{\lambda^{2/3} \text{ arc sin h}} \left[1 + 7/4\, h^2 + \frac{\Gamma(5/3)}{2\lambda^{2/3}} \right] \quad \ldots(37)$$

and $1 = 0, 1, 2, \ldots\ldots\ldots$

The presence of the magnetic field decreases the interval of t_h for which $\alpha(W, E_0, H)$ changes sign as compared with the absence of magnetic field Bonch-Bruivich & El Sharnouby, 1972).

The changes of sign in the absorption coefficient $\alpha(W, E_0, H)$ is due to the fact that after a time

$$t \simeq \frac{t_E \text{ arc sin } h}{h\{2\Pi(21 + 1)\}}$$

the a.c. field E_1 changes its direction i.e. E_0 and E_1 become opposite in direction. According to this consideration the electron gives its energy obtained from E_0 to E_1.

References

Baroff, G. A. (1964) *Phys. Rev.*, **133**, 26.

Bonch-Bruivich, V. L., and El Sharnouby, M. A. (1972) *Vest. Mosk. Univ., Fit. astron.*

Conwell, E. M. (1967) *High field transport in semiconductors*, Academic, New York.

El Sharnouby, M. A. (1972) *Vest. Mosk. Univ., Fit. astron.*, **3**, p 262.

——— (1982) Absorption coefficient of hot electrons in semiconductors. *Proc. Indian nat. Sci. Acad.*, **48A**, 203-208.

Vosylius, J. J., and Levinson, Y. B. (1966) *Zh. eksper. theor. Fiz.* **50**, 1660.

——— (1967) *Zh. eksper. theor. Fiz.*, **52**, 1013.